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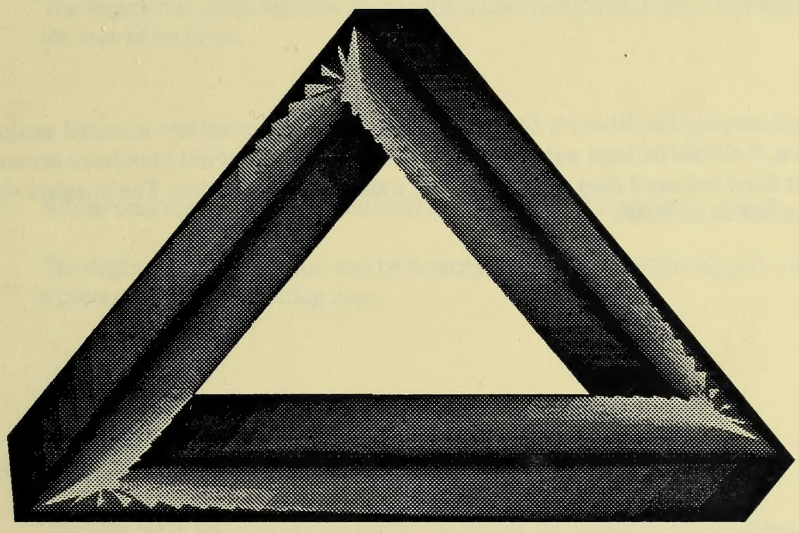

MATHEMATICS 30

LEARNING FACILITATOR'S MANUAL

UNIT 5

Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final test until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 30
Learning Facilitator's Manual
Unit 5
Quadratic Relations
Alberta Distance Learning Centre
ISBN No. 0-7741-0360-4

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Topic 1: The Physical Properties of Conic Sections

(1) 1. a. What are the degenerate conic sections?

The degenerate conic sections are a point, a line, two parallel lines, two intersecting lines, and the case of no locus.

b. Which ones can be demonstrated with a double-napped cone and a plane?

The degenerate conics which can be demonstrated on the double-napped cone include a line, a point, and two intersecting lines.

(1) 2. a. What are the nine conic sections?

The nine conic sections are the circle, the ellipse, the parabola, the hyperbola, a point, a line, two parallel lines, two intersecting lines, and the case of no locus.

b. Which ones are considered typical conic sections?

The typical conic sections are the parabola, the ellipse, and the hyperbola.

(1) 3. When you slice a right circular cone at an angle of 45°, what shape do you see?

When you slice a right circular cone at an angle of 45°, you get an ellipse. When you slice it at the vertex, you get a point.

(1) 4. How would you cut a double-napped cone to get a hyperbola?

In order to get a hyperbola, one should cut a double-napped cone such that both nappes are intersected and a vertical cut should not go through the vertical axis. If you cut through the vertical axis, you will get the degenerate case of the two intersecting lines.

(1) 5. Which of the following is a degenerate conic section?

A. a circle

B. an ellipse

C. a hyperbola

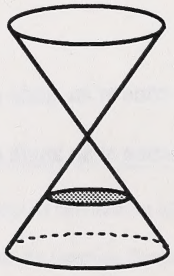
(D.) a line

The line is the case of a degenerate conic section.

(1) 6. What do you get when you rotate a right triangle about its vertical arm?

When you rotate a right triangle about its vertical arm, you get a right circular cone.

(1) 7. Draw a double-napped cone and show the intersection of a horizontal plane.



The intersection of a horizontal plane and a double-napped cone is a circle.

(1) 8. Who was the first person to use the double-napped cones for the conic sections? When and where was this done?

The first person to use the double-napped cones and intersecting plane to show the conic sections was Appolonius, a Greek mathematician. He performed the demonstration around 200 B.C. in Greece.

(1) 9. Which of the folllowing is not a degenerate conic section?

A. a point

(B.) a parabola

C. a line

D. two parallel lines

The parabola is not a degenerate conic section.

(1) 10. Which of the following is a limiting case of an ellipse?

A. the hyperbola

B. the parabola

C. two intersecting lines

(D.) the circle

The circle is a limiting case of an ellipse.

Topic 1

_____ marks

Topic 2: Describing Conics Through the Parameters

(2) 1. Name the graphs produced by the following equations.

a. $5x^2 - 5y^2 - 6x + 3y - 8 = 0$

This equation would produce a hyperbola.

b. $3x^2 + 5y^2 - 2x - 7y - 49 = 0$

This equation would produce an ellipse.

c. $9x^2 + 9y^2 - 3x - 5y - 64 = 0$

This equation would produce a circle.

d. $y^2 + 7x - 2y - 12 = 0$

This equation would produce a parabola.

(1) 2. When A and C are equal and positive; B, D, and E equal zero; and F is negative, the curve is

A. an ellipse with the longer axis along the y-axis

B. a circle with its centre in quadrant 4

C. a hyperbola with its centre at the origin

(D.) a circle with its centre at the origin

(1) 3. If either A or C is equal to zero and $B = 0$, then the curve is

A. an ellipse

B. a parabola

C. a circle

D. a hyperbola

(2) 4. Answer the following questions given the equation of a parabola $4x^2 + 9x - 5y - 25 = 0$.

a. In which direction does the parabola open?

The parabola opens up.

b. In which quadrant is the vertex of the graph located?

The vertex of the graph is in quadrant 3.

c. If the value of A were increased, what would happen to the shape of the graph?

The graph would get narrower.

d. If A were made negative, which way would the graph open?

The graph would open down.

(2) 5. Answer the following questions given the equation $x^2 + 4y^2 + 6x - 5y - 25 = 0$.

a. What type of curve does this equation represent?

It is an ellipse.

b. In what quadrant is the centre located?

The centre is in quadrant 2.

c. If the value of E were changed to $+5$, in which quadrant would the centre be located?

The centre would be in quadrant 3.

d. If the value of F were changed to -49, what change would result on the curve?

The curve would be larger.

(2) 6. Answer the following questions given the equation $x^2 + 4xy + 4y^2 - 16 = 0$.

a. What type of curve does this represent?

It represents two parallel lines.

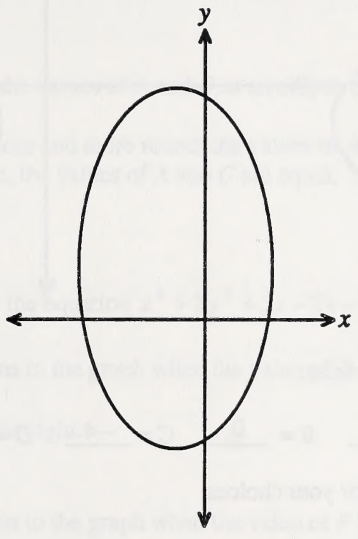
b. If the value of F were changed to -49, what would be the effect on the graph?

The lines would be further apart.

7. Write the possible values for A, B, C, D, E, and F and the reasons why you chose those values for each of the following graphs. Also, write the equation using these possible values. (Note: Although very large or very small values may be used, keep the size of the values consistent with those you have used in the practice exercises.)

(7)

a.



i. Possible values:

$A = \underline{7}$ $B = \underline{0}$ $C = \underline{2}$ $D = \underline{16}$ $E = \underline{-8}$ $F = \underline{-81}$

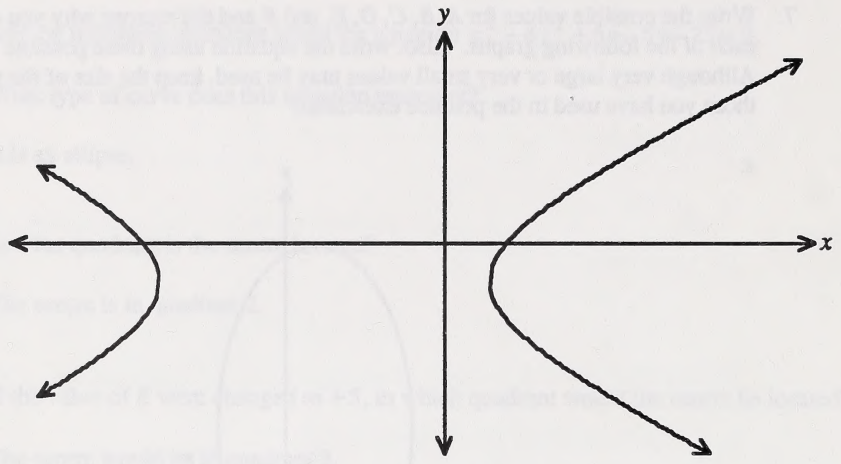
ii. Reasons for your choices:

$A > C$ since the longer axis of the ellipse is located along y-axis.
$D > 0$ and $E < 0$ since the centre is in quadrant 2.
$B = 0$ since the ellipse is vertical (no rotation).
$A, C > 0$ and $F < 0$.
All opposite signs will produce the same graph.

iii. Equation:

$$7x^2 + 2y^2 + 16x - 8y - 81 = 0$$

⑦ b.



i. Possible values:

$A = \underline{1}$ $B = \underline{0}$ $C = \underline{-4}$ $D = \underline{7}$ $E = \underline{-9}$ $F = \underline{-16}$

ii. Reasons for your choices:

$A > 0$ and $C < 0$ since the graph is a hyperbola on x-axis.
$B = 0$ since there is no rotation.
$D > 0$ since the graph is shifted to the left.
$E < 0$ since negative E values move a hyperbola on the x-axis downward.
F is a negative value.
All opposite signs will produce the same graph.

iii. Equation:

$$x^2 - 4y^2 + 7x - 9y - 16 = 0$$

(1) 8. The equation $9x^2+3y^2-4x-2y-36=0$ represents an ellipse. What value of C would make this ellipse into a circle?

Letting $C=9$ would make this into a circle.

(2) 9. Explain what happens to the values of A and C as an ellipse changes to a circle.

As the ellipse becomes more and more round, the values of A and C get closer to the same value. When the curve is a circle, the values of A and C are equal.

(2) 10. A conic is represented by the equation $x^2+2y^2+5x-2y-36=0$.

a. Describe what happens to the graph when the value of D is changed to -5.

The graph moves to the right.

b. Describe what happens to the graph when the value of F is changed to -64.

The graph becomes larger.

11. The graph of $x^2-8xy+16y^2=0$ is a degenerate conic.

(1) a. Describe the resulting graph.

The graph is a single straight line.

(4) b. Show algebraically and explain why this type of graph results.

$$x^2-8xy+16y^2=0$$
$$(x-4y)(x-4y)=0$$
$$x-4y=0 \text{ or } x-4y=0$$
$$y=\frac{1}{4}x \qquad y=\frac{1}{4}x$$

The equation factors into two factors which represent the equations of two straight lines. Since the two equations are the same, the resulting graph is only one line.

12. Answer the following questions given the equation $2x^2 + 5y^2 - 3x + 7y - 49 = 0$.

(1) a. What type of graph does this equation represent?

It represents an ellipse.

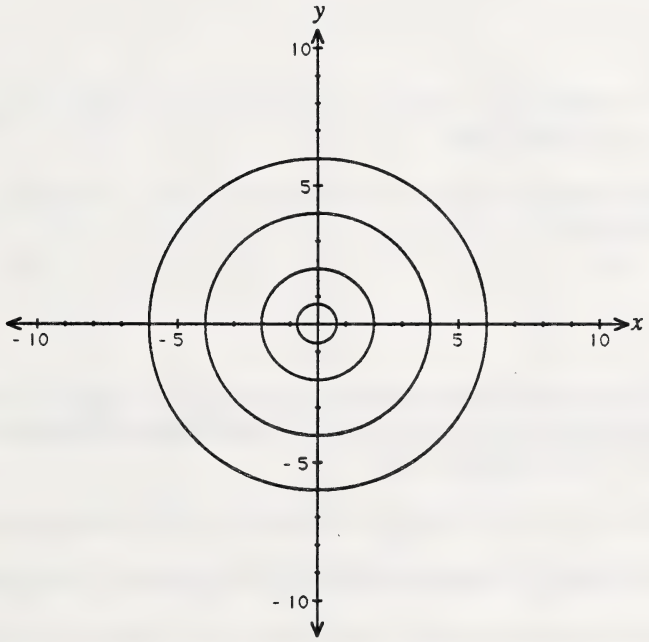
(1) b. What happens to the graph if the values of A and C are interchanged?

If A and C are interchanged, the ellipse rotates 90°. In other words, the graph becomes an ellipse with its longer axis along the y-axis.

(2) c. The centre of the graph lies in quadrant 4. Explain why.

For an ellipse negative D values move the graph to the right and positive E values move the graph down. This would put the centre in quadrant 4.

(2) 13. The following graph represents the rings of a curling house. The smallest circle is called the button and has a diameter of about 1 foot. The next circle has a diameter of 4 feet; the next one has a diameter of 8 feet; and the largest circle has a diameter of 12 feet. Explain how you could create such a pattern using the conic equations. Give the initial equation and explain what you would change in the equation to create all the circles. Also, explain why you chose the particular values for A, B, C, D, E, and F. Remember a curling house has a centre line and tee line which cross at the centre of the house.



Start with an equation of a circle like $x^2 + y^2 + F = 0$. Since the curve is a circle, you need $A = C$ and $B = 0$. To put the circle on the crossing lines or origin, you need D and E equal to zero. A small F value will give a small circle for the button. Using $F = -0.25$ will give a circle with a diameter of approximately 1 unit. The next circle has a diameter of 4 feet so you need a radius of 2 units. Using $F = -4$ will give a circle with a radius of 2 units. The next circle has a diameter of 8 feet or a radius of 4 feet so you use $F = -16$ to get the circle with a radius of 4 units. The largest circle has a diameter of 12 feet or a radius of 6 feet so you need an F value of -36.

Topic 2

_____ marks

Topic 3: Defining Each Conic as a Locus of Points

(2) 1. If you change the position of the focus relative to the directrix, what effect will be evident on the shape of the parabola?

When the distance from the focus to the directrix is increased, the parabola becomes wider. When the distance between the focus and the directrix is decreased, the parabola becomes narrower.

(2) 2. The difference in the distances from a comet on its orbit to two fixed points is constant. Describe the path of the comet.

The statement fits the locus definition for a hyperbola. Therefore, the path of the comet is hyperbolic.

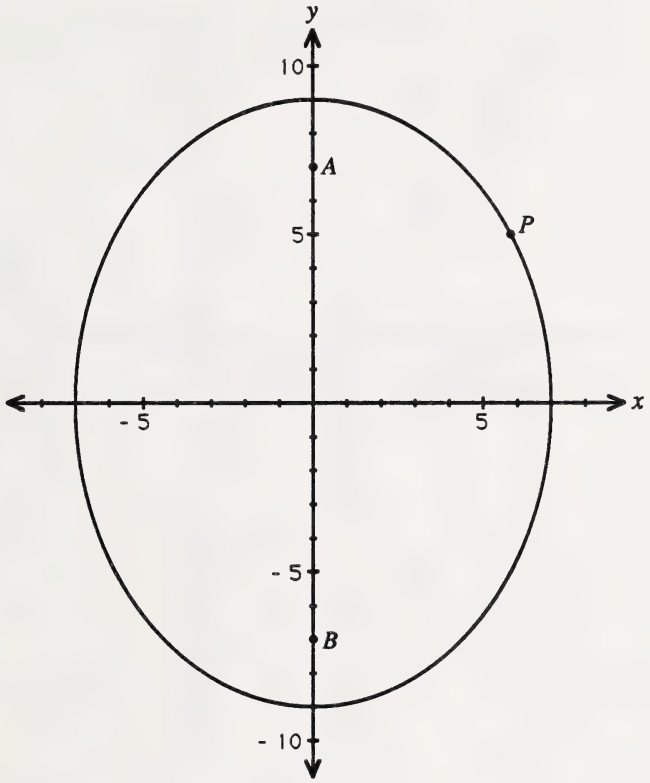
(3) 3. Use the locus definition and the string method to investigate the effect on the shape of the ellipse if the length of the string is constant but the distance between the foci is increased. Describe any restrictions which may apply.

If one focus is fixed and the other moves away, the ellipse gets narrower.

If both foci are changed, the ellipse narrows and gets more elongated.

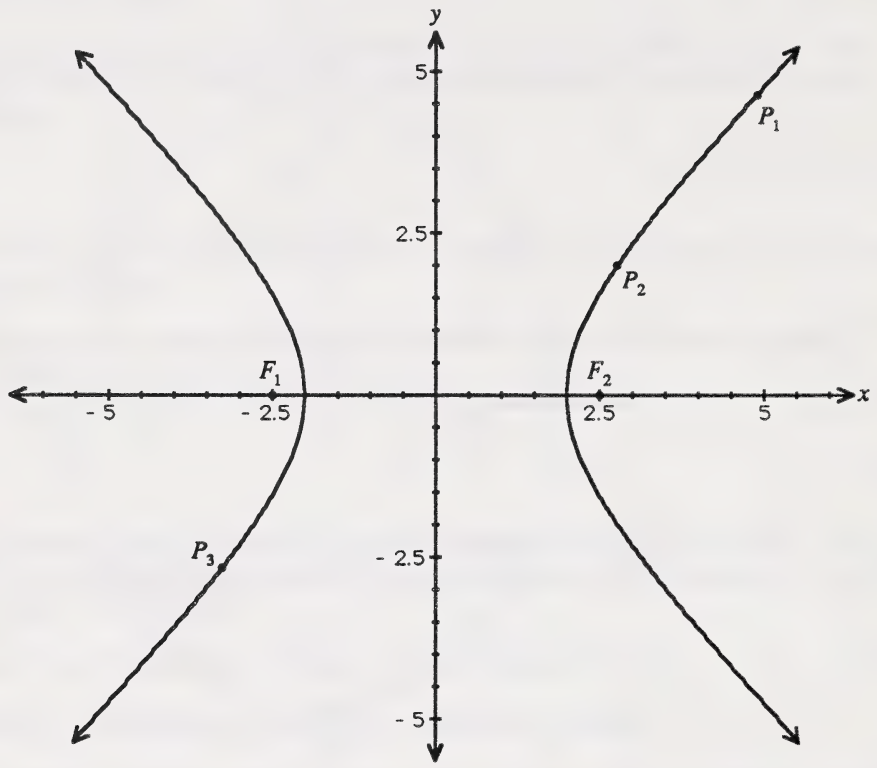
One restriction is that the distance between the foci must be less than the length of the string.

③ 4. *A* and *B* are foci and *P* is any point on the curve. Describe how you would obtain the equation for this curve.



By the locus definition for an ellipse, $d(PA)+d(PB)=k$. Using this formula and the distance formula $d=\sqrt{(x_1-x_2)^2+(y_1+y_2)^2}$, the equation for this ellipse is determined. Note that k, the constant, must be given.

③ 5. Select three points on this curve, and by measurement verify the definition of its locus.



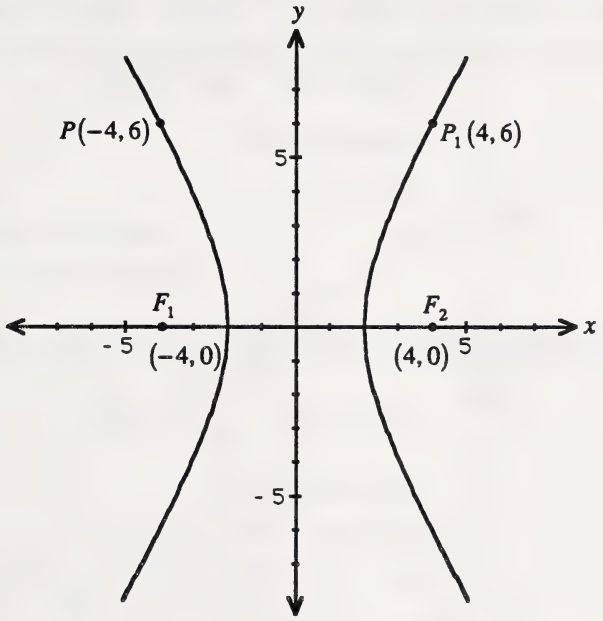
$$|d(P_1F_1) - d(P_1F_2)| = |8.9 \text{ cm} - 5.3 \text{ cm}|$$
$$= 3.6 \text{ cm}$$

$$|d(P_2F_1) - d(P_2F_2)| = |5.7 \text{ cm} - 2.1 \text{ cm}|$$
$$= 3.6 \text{ cm}$$

$$|d(P_3F_1) - d(P_3F_2)| = |2.8 \text{ cm} - 6.4 \text{ cm}|$$
$$= 3.6 \text{ cm}$$

Since the difference in distances is a constant, the locus definition holds for the hyperbola.

④ 6. $3x^2 - y^2 - 12 = 0$ is the equation of a hyperbola with foci $(-4, 0)$ and $(4, 0)$. By selecting two points on the curve, verify the locus definition for this curve.



Let $y = 6$.

$$3x^2 - (6)^2 - 12 = 0$$
$$3x^2 - 36 - 12 = 0$$
$$3x^2 = 48$$
$$x^2 = 16$$
$$x = \pm 4$$

The two points are $P_1(4, 6)$ and $P_2(-4, 6)$.

By the locus definition for a hyperbola, $\left|d(PF_1)-d(PF_2)\right|=k$.

$$\begin{aligned}\left|d(P_1F_1)-d(P_1F_2)\right| &= \left|\sqrt{[4-(-4)]^2+(6-0)^2}-\sqrt{(4-4)^2+(6-0)^2}\right| \\ &= \left|\sqrt{(8)^2+(6)^2}-\sqrt{(0)^2+(6)^2}\right| \\ &= \left|\sqrt{64+36}-\sqrt{36}\right| \\ &= |10-6| \\ &= 4\end{aligned}$$

$$\begin{aligned}\left|d(P_2F_1)-d(P_2F_2)\right| &= \left|\sqrt{[-4-(-4)]^2+(6-0)^2}-\sqrt{(-4-4)^2+(6-0)^2}\right| \\ &= \left|\sqrt{(0)^2+(6)^2}-\sqrt{(-8)^2+(6)^2}\right| \\ &= \left|\sqrt{36}-\sqrt{64+36}\right| \\ &= |6-10| \\ &= 4\end{aligned}$$

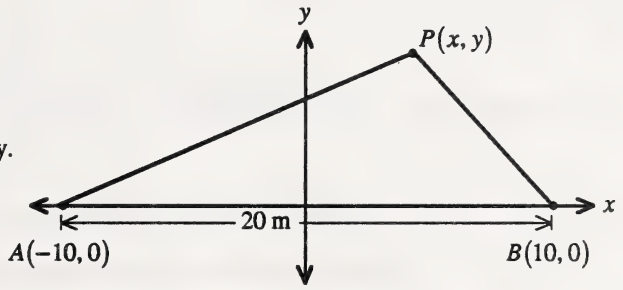
Therefore, $\left|d(PF_1)-d(PF_2)\right|=4$ and the locus definition holds for the hyperbola.

7. Two stakes are driven into the ground 20 m apart. A rope 40 m in length is fastened to the stakes. The rope runs through the ring on a guard dog's collar.

(3) a. What is the equation that describes the perimeter of an area within which the dog can move?

An ellipse is formed. By the locus definition for an ellipse $d(PA)+d(PB)=k$.

The coordinates of A and B are $(-10,0)$ and $(10,0)$ respectively.



$$d(PA)+d(PB)=40$$

$$\sqrt{[x-(-10)]^2+(y-0)^2}+\sqrt{(x-10)^2+(y-0)^2}=40$$

$$\sqrt{(x+10)^2+y^2}=40-\sqrt{(x-10)^2+y^2}$$

$$\left(\sqrt{x^2+20x+100+y^2}\right)^2=\left(40-\sqrt{x^2-20x+100+y^2}\right)^2$$

$$x^2+20x+100+y^2=1600-80\sqrt{x^2-20x+100+y^2}+x^2-20x+100+y^2$$

$$40x-1600=-80\sqrt{x^2-20x+100+y^2} \quad \text{(Divide by 40.)}$$

$$x-40=-2\sqrt{x^2-20x+100+y^2} \quad \text{(Square both sides.)}$$

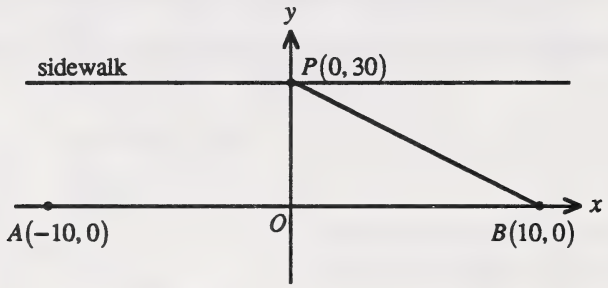
$$x^2-80x+1600=4\left(x^2-20x+100+y^2\right)$$

$$x^2-80x+1600=4x^2-80x+400+4y^2$$

$$3x^2+4y^2-1200=0$$

The equation of the ellipse is $3x^2+4y^2-1200=0$.

③ b. A sidewalk is parallel to a line through the stakes and is 30 m from this line. What is the maximum length of the rope that you can use if the dog is not to reach the sidewalk? Express your answer to the nearest whole number.



P is the furthest point on the ellipse from $\overline{AB}$. Note that $d(PO) = 30$ m and $d(OB) = 10$ m.

Therefore, the following may be stated using the Pythagorean theorem:

$$\begin{aligned} d(PB) &= \sqrt{(30)^2 + (10)^2} \\ &= \sqrt{900 + 100} \\ &= \sqrt{1000} \\ &\doteq 31.62 \end{aligned}$$

Note that $d(AP) = d(BP)$.

$$\begin{aligned} \therefore d(AP) + d(BP) &\doteq 2(31.62) \\ &\doteq 63 \end{aligned}$$

Thus, the maximum length the rope could be is approximately 63 m.

② 8. An airplane can fly a maximum of 1000 km in one day. Peace River and Athabasca are 300 km apart. The pilot always starts the trip in one city and finishes in the other. However, side trips may be taken during the flight. Within what type of figure would the pilot be confined to fly?

The pilot is confined to fly in an ellipse.

Topic 3

_____ marks

Topic 4: Using Eccentricity to Define Conics

(3) 1. How does a larger eccentricity affect the shape of a hyperbola?

If the directrix and focus remain the same, the hyperbola with a larger eccentricity would be wider and the two sections of the hyperbola would be closer together.

(3) 2. The vertical line $x=-2$ is a directrix. The fixed point $F(3,-2)$ is a focus. The eccentricity is 5. Find the equation of the conic.

$$\frac{\sqrt{(x-3)^2+(y+2)^2}}{x+2}=5$$

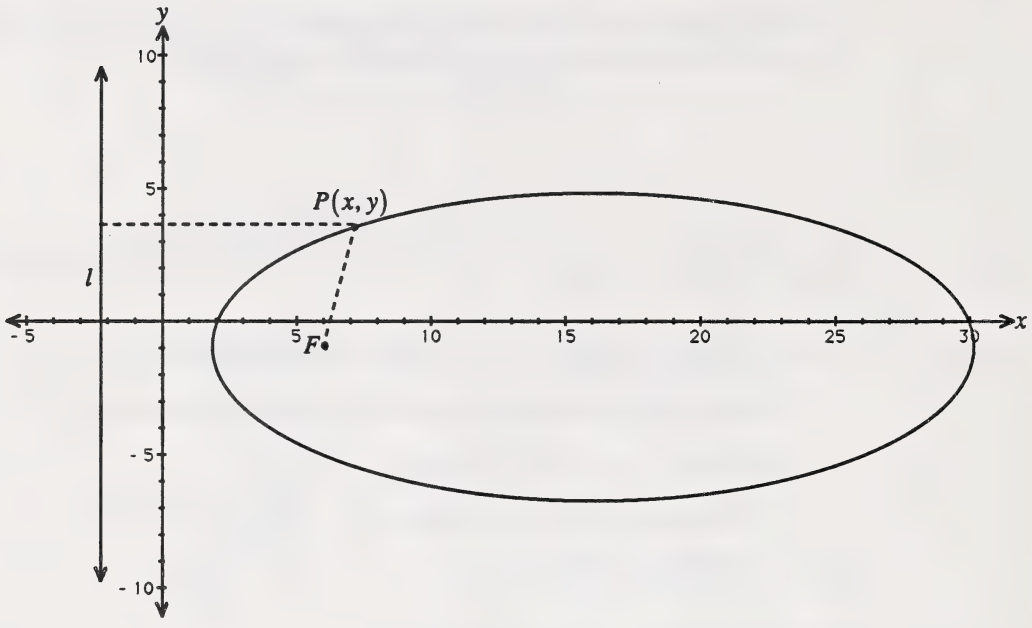
$$x^2-6x+9+y^2+4y+4=25\left(x^2+4x+4\right)$$

$$x^2+y^2-6x+4y+13=25x^2+100x+100$$

$$24x^2-y^2+106x-4y+87=0$$

The equation is $24x^2-y^2+106x-4y+87=0$.

③ 3. Determine the eccentricity of the following ellipse. Note that F is the focus and l is the directrix. Give your answer to one decimal place.



$$e = \frac{2 \text{ cm}}{3.9 \text{ cm}} \doteq 0.5$$

Thus, the eccentricity is $\frac{1}{2}$.

(12) 4. Define the four conics in terms of eccentricity.

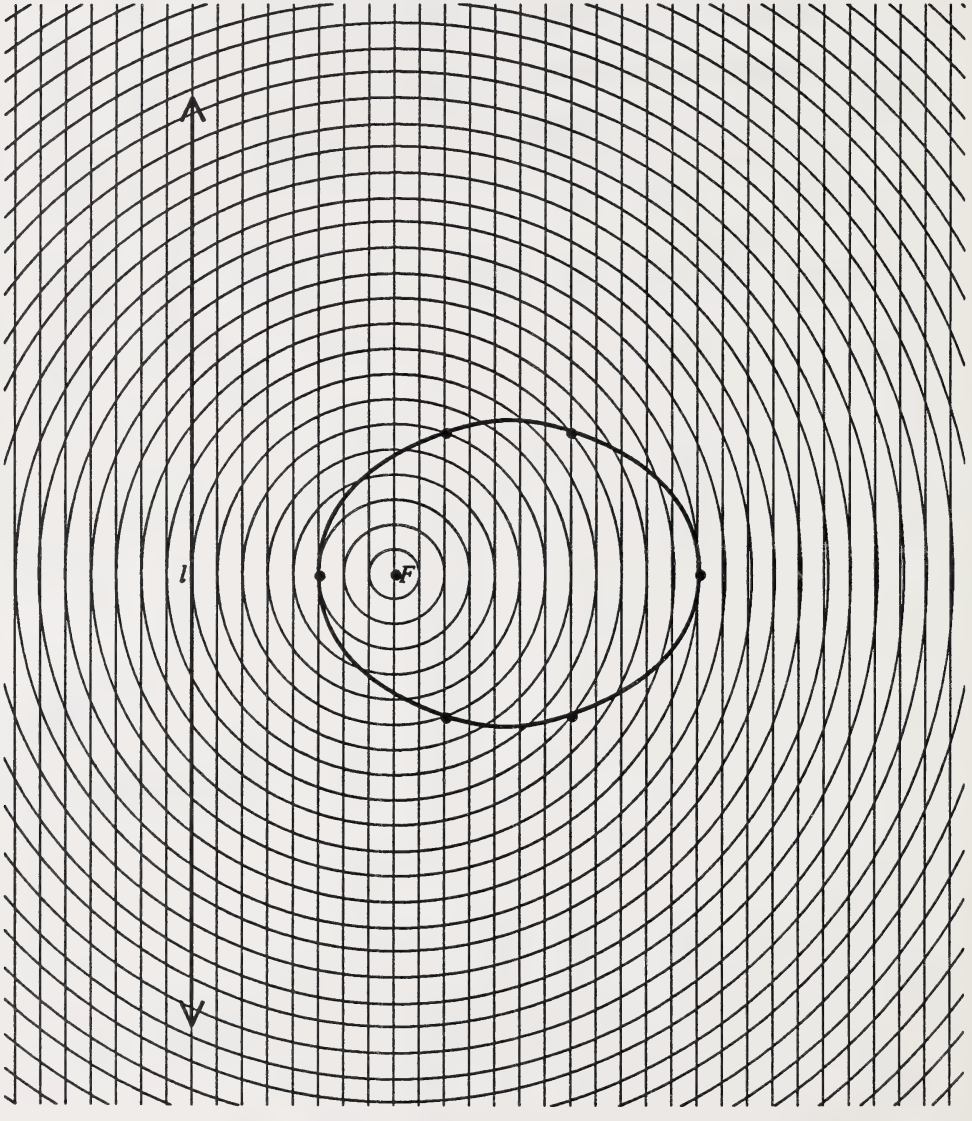
Circle: The limiting position of an ellipse is a circle as the eccentricity approaches 0.

Parabola: The locus of points is a parabola when $e = 1$.

Ellipse: The locus of points is an ellipse when $0 < e < 1$.

Hyperbola: The locus of points is a hyperbola when $e > 1$.

④ 5. The distance between a fixed line and a fixed point is 8 units. The ratio of the distance between any point and the fixed point to the distance between the same point and the fixed line is $\frac{3}{5}$. Use this circle-line graph to sketch the conic.



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(REVISED 1991)
SECOND EDITION
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